**Homework 3** Due October 22, 2021

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# Problem 3.1

Five people get on the elevator that stops at five floors. In how many ways they can get off? For example, one person gets off at the first floor, two will get off at the third, and the remaining two at the fifth floor. In how many ways they can get off at different floors? Now, consider that people in elevator have names, say *A, B, C, D,* and *E*, assuming that, for example, the case *A* on the first floor is different from the case *B* on the first floor. Answer the previous questions with this assumption.

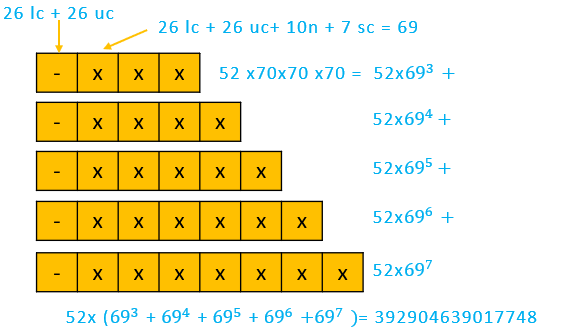
# Problem 3.2

A lady wishes to color her fingernails on one hand using at most two of the colors: red, yellow, and blue. In how many ways she can do it?

# Problem 3.3

On a computer system the valid password should start with a letter (26 letters at all), it is case sensitive (upper case is different form lowercase), its length is from 4 up to 8, and symbols are

*{*1,2,3,..,9,0,a,b,c,..z,A,B,C,..,Z,?,!,#,%,&,\*,/*}*. How many such passwords can be created?



# Problem 3.4

How many of the billion numbers in the range from 1 to 109 contain the digit 1?

# Problem 3.5

Solve the following problems using the pigeonhole principle. For each problem, try to identify the pigeons, the pigeonholes, and a rule assigning each pigeon to a pigeonhole.

1. In every set of 100 integers, there exist two whose difference is a multiple of 37.
2. For any 5 points inside a unit square (not on the boundary), there are 2 points at distance less than 1 .

*√*

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*The pigeons are the points. The pigeonholes are the four subsquares of the unit square, each of side length 1/2. Pigeons are assigned to the subsquare that contains them, except that if the pigeon is on a boundary, it gets assigned to the leftmost and then lowest possible subsquare that includes it (so the point at (1/2, 1/2) is assigned to the lower left subsquare).*

*There are five pigeons and four pigeonholes, so more than one point must be in the same subsquare.*

*The diagonal of a subsquare is 1/ √2, so two pigeons in the same hole are at most this distance.*

*But pigeons must be inside the unit square, so two pigeons cannot be at the opposite ends of the same subsquare diagonal.*

*So at least one of them must be inside the subsquare, so their distance is less than the length of the diagonal.*

1. Show that if *n* + 1 numbers are selected from 1*,* 2*,* 3*, . . . ,* 2*n* , two must be consecutive, that is, equal to *k* and *k* + 1 for some *k*.

*{ }*

# Problem 3.6

Here are the solutions to the next 8 questions, in no particular order.

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*1. 2. 3. 4. 5. 6. 7.*

Answer all the questions. Justify your answers.

* 1. How many solutions over the natural numbers are there to the inequality

*x*1 + *x*2 + *. . .* + *xn ≤ m* ?

* 1. How many length *m* words can be formed from an *n* letter alphabet, if no letter is used more than once?

*−*

* 1. How many length *m* words can be formed from an *n* letter alphabet, if letters can be reused?

*−*

* 1. In how many ways you can connect elements from set *A* to elements from set *B* when *A* = *m* and

*| |*

*B* = *n*? One element from *A* can be connected to as many elements from *B* as possible.

*| |*

* 1. How many injections are there from set *A* to set *B*, where *A* = *m* and *B* = *n,* and *n > m*?

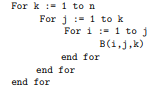
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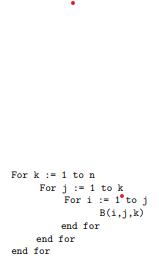
* 1. How many ways are there to place a total of *m* **distinguishable** balls into *n* **distinguishable** urns, with some urns possibly empty or with several balls?
  2. How many ways are there to place a total of *m* **indistinguishable** balls into *n* **distinguishable** urns, with some urns possibly empty or with several balls?
  3. How many ways are there to put a total of m distinguishable balls into n distinguishable urns with at most one ball in each urn?

# Problem 3.7

**Bonus Problem (for extra credit)**

How many times will the code *B* in the following program be executed?



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